



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$1-p=1-2^4/6^3=1/1001176888=P$, chance that D will throw less than 52.

P^2 =chance that D and E will both throw less.

$\therefore aP^2=C$'s expectation on the supposition that C wins and no ties.

49. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

A square whose side is $2a$ and an equilateral triangle whose altitude is $3a$ are fastened together at their centers, but otherwise free to move. If they are thrown on a floor at random, what is the average area common to both?

Solution by HENRY HEATON, M. Sc., Atlantic, Iowa, and the PROPOSER.

In the figure let O be the common center of the square and the triangle.

Then $OK=ON=OH=OM=OL=OP=OI=a$.

Let the triangle $KON=2\theta$.

Then $\angle NOH=\frac{1}{2}\pi-2\theta$, $\angle HOM=\frac{3}{2}\pi-(\frac{1}{2}\pi-2\theta)=\frac{1}{2}\pi+2\theta$, $\angle MOL=\frac{1}{2}\pi-(\frac{1}{2}\pi+2\theta)=\frac{1}{2}\pi-2\theta$, $\angle LOP=\frac{3}{2}\pi-(\frac{1}{2}\pi-2\theta)=\frac{1}{2}\pi+2\theta$, and $\angle POI=\frac{1}{2}\pi-(\frac{1}{2}\pi+2\theta)=\frac{1}{2}\pi-2\theta$.

Area of surface, $KONQ$, $=a^2 \tan \theta$;

area of surface, $NOHW$, $=a^2 \tan(\frac{1}{2}\pi-\theta)$;

area of surface, $HOMV$, $=a^2 \tan(\frac{1}{2}\pi+\theta)$;

area of surface, $MOLU$, $=a^2 \tan(\frac{1}{2}\pi-\theta)$;

area of surface, $KOPT$, $=a^2 \tan(\frac{1}{2}\pi+\theta)$;

area of surface, $POIS$, $=a^2 \tan(\frac{1}{2}\pi-\theta)$;

area of square, $OKDI$, $=a^2$.

Hence the area common to the square and triangle is

$$S=a^2[1+\tan\theta+\tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\tan(\frac{1}{2}\pi-\theta)].$$

The positions for $\theta > \frac{1}{2}\pi$ are exact repetitions of those for $\theta < \frac{1}{2}\pi$.

Hence the required average area is

$$A=\int_0^{\frac{1}{2}\pi} Sd\theta \div \int_0^{\frac{1}{2}\pi} d\theta = \frac{12a^2}{\pi} \int_0^{\frac{1}{2}\pi} \left[1+\tan\theta+\tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\right.$$

$$\left. \tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\tan(\frac{1}{2}\pi-\theta) \right] d\theta = a^2 \left[1 + \frac{12}{\pi} \log_e 2 \right].$$

This problem was also solved in a very excellent manner by G. B. M. Zerr.

50. Proposed by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Find (1), the average length of all straight lines having a given direction, between 0 and a ; (2), the average length of chords drawn from one extremity of the diameter a of a semi-circle to all points in the semi-circumference; and (3), find the average area of all triangles formed by a straight line of constant length a sliding between two straight lines at right angles.

